Reaction-diffusion equations are often used to model the dispersal of genes or organisms. In this talk, we replace the diffusion term with a nonlocal dispersal term; that is, we study the model

$$u_t = \rho \left(\int_\Omega \beta(x,y)u(y)\,dy - u \right) + f(u) \tag{1}$$

We hope to better understand the long-term dynamics of this model by identifying the ω-limit sets and stationary solutions. This first obstacle to overcome in this problem is to prove that orbits are precompact, so that limits exist. Even then, the general theory of parabolic PDE's is not very helpful in proving things about the ω-limit sets. Though we will spend some time discussing the case for general β, the problem is much more tractable when $\beta \equiv 1$. We will discuss the results for this simpler case, and then discuss the possibilities for generalizing these results for other kernels β. In the case when $\beta \equiv 1$, we prove that all solutions tend towards a single equilibrium.

This work originally came out of the work by Hutson, Martinez, Mischaikow and Vickers to understand the evolution of dispersal rates for the model (1).