Wave power flux in inhomogeneous media

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Power flux in an inhomogeneous transmission line (TL) such as the cochlea is fundamentally important to our understand of the cochlear amplifier. A TL's characteristic impedance $z_0(x)$ is required for the basic definition of wave power and reflectance. However it seems that there is a fundamental lack of agreement on the definition of $z_0(x)$ for inhomogeneous lines. Several classic books even claim that the characteristic impedance does not generalize beyond its standard use in homogeneous 1-dimensional TLs. There are two accepted ways to define z_0: (1) the ratio of the voltage to current (or pressure over velocity) for a wave launched in one direction (i.e., the input impedance), and (2) $z_0(x) = \sqrt{Z(x,s)/Y(x,s)}$, where $Z(x,s) = sL(x)$ is the series impedance, $Y(x,s) = sC(x)$ is the shunt admittance, each per unit length, and $s = i\omega$ is the complex radian frequency. We shall show how these two definitions are related to forward and backward power flux. We shall show that if one defines the forward and backward waves in a certain manner, then the two definitions are the same. However these solutions are not the standard ones given in the text books and journal articles. A closely related set of questions exists for the "propagation function," $\gamma(x,s) = \sqrt{Z(x,s)Y(x,s)}$ due to its similarity to definition (2) for $z_0(x)$.