Above image shows the heart in diastole and systole.

- **Diastole**
  - Ao: Aorta
  - LA: Left Atrium
  - LV: Left Ventricle

- **Systole**
  - Ao: Aorta
  - LA: Left Atrium
  - LV: Left Ventricle

Below the diagrams is a graph showing pressure ($P$) against volume ($V$). The graph illustrates the relationship between pressure and volume during the heart cycle. The pressure is divided into two parts:

- $P_{Ao}$: Pressure in the Aorta
- $P_{LA}$: Pressure in the Left Atrium

The graph also highlights a change in volume ($\Delta V$) as the heart contracts and relaxes.
The diagram illustrates the pressure and force changes during systole and diastole.

- **Ao**: Represents the aortic pressure.
- **LV**: Represents the left ventricular pressure.
- **LA**: Represents the left atrial pressure.

The graph shows:
- **Diastole**: A period of relaxation where the heart fills with blood.
- **Systole**: A period of contraction where the heart pumps blood.

The graphs also indicate:
- **Mi**: Represents the myocardial force.
- **“Lub” “Dup”**: Represents the auscultatory sounds associated with cardiac valves.
maps – to morphologists, and provide them with some data they could compare to their own. The correspondence with microdissected specimens is almost direct for the sub-epicardial fibres, which have been well studied and illustrated by Anderson et al. (1980), Greenbaum et al. (1981), and Fernandez-Teran and Hurle (1982). For example, there is a striking resemblance between the maps of the diaphragmatic face of the heart (Fig. 3 bottom) and the images provided by these authors. For the deepest fibres, the correspondence is indirect, but once the full significance of fibre elevation and azimuth has become clear, the pictures are mostly self-explanatory, and provide new insights into some of the more complicated areas of the ventricular mass.

The second aim was to give an account of the overall architecture of the fibres in the fetal heart at mid gestation. Our studies show that the “flattened-rope analogy” of Torrent-Guasp (1975) and Streeter (1979) is inadequate to describe the fibre architecture at this developmental period. The inconsistencies of this model have already been discussed by Lunkenheimer et al. (1997). But, at least during the fetal period, one major inconsistency obliges us to discard it. In this model, the purportedly continuous band stretched between the two outflow tracts is said to follow a preferential pathway that, at the level of the base of the diaphragmatic face of the ventricular mass, is located between the fibres of the right ventricle and the left ventricle (Fig. 1B). This is not the preferential pathway observed in our study, where the latitudinal fibres of the diaphragmatic face of the right ventricle merge preferentially with the latitudinal fibres of the septum.

The description of the ventricular mass based on layers is less inadequate than the “flattened rope analogy”, but it, too, gives an excessive simplification of the fibre architecture. This is mostly because, as shown in our results, the fibres of the ventricular walls and the septum run constantly from one layer to the other. The azimuth
Aortic Leaflet Stained for Collagen
A.A.H.J. Sauren
Aortic Valve

Closed

Open
AORTIC VALVE

CLOSED VALVE VIEWED FROM AORTA

AORTA

AXIAL CROSS SECTIONS OF CLOSED VALVE

LEFT VENTRICLE
AORTIC VALVE

CLOSED VALVE VIEWED FROM AORTA

AORTA

AXIAL CROSS SECTIONS OF CLOSED VALVE

LEFT VENTRICLE
UNIFORM PRESSURE LOAD SUPPORTED BY ONE-PARAMETER FAMILY OF FIBERS UNDER TENSION:

\[ T \cdot X_u \, du \]

\[ u = \text{arc length along fiber} \]

\[ u = u_2 \]

\[ u = u_1 \]

\[ -T \cdot X_u \, du \]

\[ T(u) \, du = \text{fiber force} \]

\[ P_0 = \text{applied pressure load} \]
\[(TX_u)\bigg|_{u_1}^{u_2} \, dr + \int_{u_1}^{u_2} p_0(X_u \times X_v) \, du \, dv = 0\]

\[\int_{u_1}^{u_2} \left[ (TX_u)_u + p_0(X_u \times X_v) \right] \, du = 0\]

\[\left( Tu \cdot X_u \right) + \left( TX_{uu} + p_0(X_u \times X_v) \right) = 0\]

tangential \quad normal

It follows that:

- \( T = T(v) \), independent of \( u \).
- Fibers are geodesics.
- Coordinates form an orthogonal net (provided that \( u=0 \) is chosen orthogonal to fibers).
ELIMINATE MECHANICAL VARIABLES:

Let \( dV = \frac{T(u) \, dv}{r_0} \)

Then \( X_{uu} + (X_u \times X_V) = 0 \)

Apply \( X_u \times \) to solve for \( X_V \):

\[
X_V = X_u \times X_{uu}
\]
**Boundary Conditions**

\[ u = u_0(v) \]

\[ u = -u_0(v) \]

\[ u = 0 \]

*At the commissures* \( u = \pm u_0(v) \):

\[ 0 = \frac{d}{dv} X(\pm u_0(v), v) \]

\[ = \pm u_0'(v) X_u + X_v \]
Since $X_u$ and $X_v$ are orthogonal and $X_u$ is a unit vector, we have

$u_0^3(V) = 0 \Rightarrow u_0$ is independent of $V$, that is, all fibers are the same length!

and

$X_v(\pm u_0, V) = 0$

$\Downarrow$

$(X_u \times X_{uu})(\pm u_0, V) = 0$

$\Downarrow$

$X_{uu}(\pm u_0, V) = 0$

i.e., fibers at commissures are straight.
Numerical Scheme (T. Buttke)

\[ j = \frac{1}{2} \quad \Delta u = \frac{2u_0}{N} \]

Let \( \zeta = X u \) (unit tangent). Then

\[ \zeta_V = \zeta \times \zeta u u \]

\[ \frac{\zeta_j^{n+1} - \zeta_j^n}{\Delta V} = \sigma_j^n \times \frac{\sigma_{j+1}^n - 2\sigma_j^n + \sigma_{j-1}^n}{(\Delta u)^2} \]

where \( \sigma_j^n = \frac{\zeta_j^n + \zeta_j^{n+1}}{2} \)

and \( j = \frac{1}{2}, \frac{3}{2}, \ldots, (N-\frac{1}{2}) \)
Boundary conditions are enforced by reflection:

\[ \tau^n_{-\frac{1}{2}} = \tau^n_{\frac{1}{2}}, \quad \tau^n_{N+\frac{1}{2}} = \tau^n_{N-\frac{1}{2}} \]

Equations for \( \tau^{n+1}_{\frac{1}{2}} \ldots \tau^{n+1}_{N-\frac{1}{2}} \) are solved by fixed-point iteration, which converges if

\[ \frac{\Delta V}{(\Delta u)^2} \leq \frac{1}{4} \]
After solving for $\mathcal{I}^n$, we construct the $n^{th}$ discretized fiber as

$$X_k^n = X_0 + (\Delta u) \left( \mathcal{I}_2^n + \cdots + \mathcal{I}_{k-\frac{1}{2}}^n \right)$$

for $k = 1 \ldots N$. Here, $X_0$ is one commisural point and $X_N$ is the other one, so it is important that $X_N^n$ should be independent of $n$. This can be proved from our scheme with its reflecting boundary conditions.
INITIAL CURVE

\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]

\[ \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \]

\[ \left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right) \]

\[ z = -c x(x-\frac{1}{2})(x-1) \]
Numerical Solution of the Fiber Architecture Equations of the Aortic Valve. David M. McQueen & Charles S. Peskin
Hog ventricles

These drawings represent the ventricles before any muscle fascicles were removed.

2. Anterior view.
3. Posterior view.
4 Apical view. This drawing represents the ventricles before any muscle fasicles were removed. The upper edge of the drawing is the anterior surface of the heart.

5 Anterior view. The outer layer was removed and the interventricular band was transected. Then the ventricles were separated along the anterior sulcus. Note the penetration of the right septal fasicles by the left septal fasicles.
6. Posterior view. This drawing represents a more advanced stage of dissection of the heart in figure 5: (1) The interventricular and intraventricular bands were removed. (2) The fibrous base of the right ventricle was separated from the fibrous base of the left ventricle. This included the transection of the longitudinal muscle of the right ventricle. (3) The origins of some of the right inner, right septal, and left septal fascicles were transected from the medial portion of the aortic ring. The line extending from the pulmonary to the right A-V orifice formed part of the medial portion of the aortic ring before the ventricles were separated and represents the origin of these transected fascicles. (4) All the right septal band, except the intraseptal portion, was removed. (5) The ventricles were separated along the posterior sulcus.

7. An isolated intact cylinder. (The mode of isolation is explained in the text.)
Figure 2. Cardiac muscle cells are grouped in fibers and are connected at intercalated discs.
within 5 mm of a given sampling point. Shrinkage of the preparation due to dehydrating, embedding, and sectioning by microtome was ap-

proximately 18% in the fiber plane parallel to the epicardium.

Results

An example of the change of fiber angle through the wall at a single sampling point in the left ventricle is shown in Figure 3 by

Fiber angles for four sampling sites, a through d, in a T-top section from a heart in diastole are plotted as a function of percent wall thickness. Zero percent of wall thickness implies the endocardial surface. M represents the mean of the data at these four sites.

Circulation Research, Vol. XXIV, March 1969
Fiber Architecture of Left Ventricle

1) Equilibrium: \[ \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} = 0 \]

2) Fiber stress: \[ \sigma_{ij} = -p \delta_{ij} + T \mathbf{e}_i \mathbf{e}_j, \quad |T|=1 \]

3) Constant-area fiber tubes: \[ \nabla \cdot \mathbf{Z} = 0 \]

4) Axial symmetry:

5) Thin wall.
EQUATION OF EQUILIBRIUM

\[ \nabla p = T(T \cdot Vr) + I V \cdot (T \dot{r}) \]

\[ = TK \eta + I V \cdot (T \dot{r}) \]

**principal normal**

**curvature**

**Special case:** \[ D \cdot (T \dot{r}) = 0 \]

\[ I \cdot Vp = 0 \Rightarrow \text{fibers run on } p=\text{constant} \]

\[ \nabla p = TK \eta \Rightarrow \text{fibers are geodesics on } p=\text{constant} \]
$P = 0$  $P = P_0$  $P = 0$

- Endocardial + Epicardial wall surfaces.
- Surfaces of constant pressure
- Fiber surfaces
**THEOREM:** If fibers are geodesic, they run (alas) along \( p = \text{constant} \!\)

**If:**
- Egn. of equilibrium
- Axial symmetry

Arrow pointing at:
- Fibers geodesic on “fiber surfaces”
- Non-trivial Swirl: \( \mathbf{I} \cdot \hat{\mathbf{\Omega}} \neq 0 \)

**Then:**
\[
\nabla \cdot (\mathbf{F} \mathbf{F}) = 0
\]
Fiber surfaces are \( p = \text{constant} \)

**Proof:**
\[
(\hat{\mathbf{\Omega}} \cdot \nabla p) = TK(\hat{\mathbf{\Omega}} \cdot \mathbf{n}) + (\hat{\mathbf{\Omega}} \cdot \mathbf{I}) \nabla \cdot (\mathbf{F} \mathbf{F})
\]
- Zero if fibers are geodesic (on any surface of revolution.)
- Zero by axial symmetry
Curvilinear coordinates for asymptotic analysis:

\[ X(u, v, \theta) = (R(u, v) \cos \theta, R(u, v) \sin \theta, Z(u, v)) \]

where

\[ R(u, v) = R_0(u) + \varepsilon v Z'_0(u) \]

\[ Z(u, v) = Z_0(u) - \varepsilon v R'_0(u) \]

\[ (R'_0)^2 + (Z'_0)^2 = 1 \]
Choice of variables for asymptotic analysis:

\[ p = P(u, v) \]

\[ \varepsilon T = S(u, v) \]

\[ \tau = \tau_u(u, v) \frac{\partial x}{\partial u}(u, v, \theta) \]

\[ + \tau_v(u, v) \frac{\partial x}{\partial v}(u, v, \theta) \]

\[ + \tau_\theta(u, v) \frac{\partial x}{\partial \theta}(u, v, \theta) \]
where
\[ p, S, r_u, r_v, r_\theta = O(1) \]

Note that
\[ T = O(\varepsilon^{-1}) \]
which is needed to support steep pressure gradient in thin wall

\[ \frac{\partial X}{\partial \nu} = O(\varepsilon) \]
which implies that fibers are approximately parallel to middle surface \( \nu = 0 \)
The $\varepsilon^0$ equations:

\[ 1 = \tau_u^2 + \tau_0^2 R_0^2 \]

\[ 0 = \frac{\partial}{\partial u} (R_0 \tau_u) + \frac{\partial}{\partial \tau} (R_0 \tau_0) \]

\[ 0 = D(S \tau_u) - S \tau_0^2 R_0 R_0' \]

\[ \frac{\partial P}{\partial \tau} = -S \left[ \tau_u^2 (R_0' \tau_0'' - \tau_0' R_0'') + \tau_0^2 R_0 \tau_0' \right] \]

\[ 0 = [D(S \tau_0)] R_0^2 + 2S \tau_u \tau_0 \tau_0' R_0' \]

where

\[ D = \tau_u \frac{\partial}{\partial u} + \tau_0 \frac{\partial}{\partial \tau} \]
Invariants on the fibers
(and hence by symmetry on the fiber surfaces):

By combining the 1st, 3rd, & 5th of the $E^0$ equations, we find

$$D(S) = 0$$

$$D(\hat{\Theta} R_0^2) = 0$$

Note that $\hat{\Theta} R_0 \sim \tau \cdot \hat{\Theta} = : \cos \alpha$

so we have

$$D(R_0 \cos \alpha) = 0$$
Here $\alpha$ is the angle between the fiber direction and the circumferential direction, so the result that $R_0 \cos \alpha$ is constant on a fiber shows that the fiber is a geodesic on the middle surface. (Clairaut)

But this result holds only in the limit $\varepsilon \to 0$, and we have seen above that the exact equation rule out geodesic paths for the fibers on the fiber surfaces.
Introduction of a Stream Function:

Since

\[ \frac{\partial}{\partial u} (R_0 \tau_u) + \frac{\partial}{\partial v} (R_0 \tau_v) = 0 \]

\( \exists \, \psi(u,v) \) such that

\( R_0 \tau_u = \frac{\partial \psi}{\partial v} \)

\( R_0 \tau_v = -\frac{\partial \psi}{\partial u} \)

Note that \( D\psi = 0 \), so \( \psi \) is another invariant of a fiber.
Self-similar solution of the $\varepsilon^0$ equations:

Seek $f()$ such that

$$\sin \alpha = \tau_u = f'(\frac{v}{R_0(u)}) = f'(V)$$

Where

$$f'(0) = 0$$

Since $\tau_u = 0$ defines the middle surface $v = 0$

but $f'(V) \neq 0$, since that would mean circumferential fibers only. (Also, we may set $f'(0) = 0$, since only $f'$ matters.)
We have
\[ \frac{\partial \psi}{\partial u} (u, v) = R_0(u) f'\left( \frac{v}{R_0(u)} \right) \]
\[ \psi(u, v) = R_0^2(u) f\left( \frac{v}{R_0(u)} \right) + g(u) \]

Also
\[ \sin^2 \alpha + \cos^2 \alpha = 1 \]
\[ (f''(V))^2 + \frac{J(\psi)}{R_0^2(u)} = 1 \]

Since \( R_0 \cos \alpha \) is constant on a fiber and therefore a function of \( \psi \).
Therefore,

\[
(f'(V))^2 + \frac{J(R^2_0(u)f(V)+g(u))}{R^2_0(u)} = 1
\]

From now on, treat \(u, V\) as independent variables. Apply \(\frac{\partial}{\partial V}\):

\[
2f'(V)f''(V) + f'(V)J'(R^2_0(u)f(V)+g(u)) = 0
\]

and then \(\frac{\partial}{\partial u}\):

\[
J''(2R_0(u)R'_0(u)f(V)+g'(u)) = 0
\]
But 

\[ 2 R_0(u) R_0'(u) f(V) + g'(u) = 0 \]

\[ \Rightarrow f(V) = -\frac{g'(u)}{2 R_0(u) R_0'(u)} \]

\[ \Rightarrow f'(V) = 0 \]

Contrary to hypothesis, so the only possibility is

\[ J''(\Psi) = 0 \Rightarrow J(\Psi) = a \Psi + b \]

where \( a, b \) are constant.
Now with $J(\psi)$ determined, it is easy to show that
\[ f(V) = -\frac{aV^2}{4} \]
\[ \psi(u, N) = -\left(\frac{a}{4}u^2 + \frac{R_0^2(0) - R_0^2(u)}{a}\right) \]
where the constant $b$ has been chosen to make $\psi(0) = 0$. 
The fiber angle $\alpha$ is therefore given by

$$\sin \alpha = -\frac{a}{2} \frac{\nu}{R_0(u)}$$

or

$$\alpha = -\arcsin \left( \frac{a}{2} \frac{\nu}{R_0(u)} \right)$$

The domain of $\nu$ is therefore restricted by

$$|\nu| \leq \frac{2R_0(u)}{|a|}$$

At the extremes of $\nu$, $\alpha = \pm \frac{\pi}{2}$, and these are fiber surfaces, since on them $\gamma$ is constant.
Qualitative behavior of the fiber surfaces:

Choose the origin of \( u \) so that \( R_0(u) \) is maximized by \( u = 0 \). Then \( \Psi(u,v) \) has a maximum (\( a > 0 \)) or minimum (\( a < 0 \)) at \( u = v = 0 \). In either case, the fiber surfaces in the neighborhood of \( u = v = 0 \) are nested tori.
Details are in the following papers:

**Peskin CS and McQueen DM:**
Mechanical equilibrium determines the fractal fiber architecture of the aortic heart valve leaflets. American Journal of Physiology 266: H319-H328, 1994
[http://ajpheart.physiology.org/content/266/1/H319](http://ajpheart.physiology.org/content/266/1/H319)

**Stern JV and Peskin CS:**
[http://dx.doi.org/10.1142/S0218348X9400065X](http://dx.doi.org/10.1142/S0218348X9400065X)

**Peskin CS:**
[http://dx.doi.org/10.1002/cpa.3160420106](http://dx.doi.org/10.1002/cpa.3160420106)